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Semiparametric single-index assumptions are widely used dimension reduction approaches that represent a convenient compromise between the parametric and fully nonparametric models for regressions or conditional laws. In a mean regression setup, the SIM assumption means that the conditional expectation of the response given the vector of covariates is the same as the conditional expectation of the response given a scalar projection of the covariate vector. In a conditional distribution modeling, under the SIM assumption the conditional law of a response given the covariate vector coincides with the conditional law given a linear combination of the covariates. In this paper, a novel kernel-based approach for testing SIM assumptions is introduced. The covariate vector needs not have a density and only the index estimated under the SIM assumption is used in kernel smoothing. Hence the effect of high-dimensional covariates is mitigated while asymptotic normality of the test statistic is obtained. Irrespective of the fixed dimension of the covariate vector, the new test detects local alternatives approaching the null hypothesis slower than $n^{-1/2}h^{-1/4},$ where h is the bandwidth used to build the test statistic and n is the sample size. A wild bootstrap procedure is proposed for finite sample corrections of the asymptotic critical values. The small sample performance of our test is illustrated through simulations.