A Geometric Approach to Density Estimation with Additive Noise

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Abstract
We introduce and study a method for density estimation under an additive noise model. Our method does not attempt to maximize a likelihood, but rather is purely geometric: Heuristically, we $L_2$-project the observed empirical distribution onto the space of candidate densities that are reachable under the additive noise model. Our estimator reduces to a quadratic program, and so can be computed efficiently. In simulation studies, it roughly matches the accuracy of fully general maximum likelihood estimators at a fraction of the computational cost. We also give a theoretical analysis of the estimator, and show that it is consistent, attains a quasi-parametric convergence rate under moment conditions, and is robust to model mis-specification. We provide an R implementation of the proposed estimator in the package nlpden.

Keywords: M-estimator, minimum distance estimator, mixture model, quadratic program, shape constrained estimator.

1 Introduction

Consider the high-dimensional Gaussian noise model, in which we observe

$$X_i = \mu_i + \varepsilon_i, \varepsilon_i \sim \mathcal{N}(0,1) \text{ independently for } 1 \leq i \leq n. \tag{1}$$

Following Robbins [1964], this model has often been analyzed from an empirical Bayes perspective. In a classical Bayesian setting, we assume that $\mu_i \sim G$ for some prior distribution $G$. 