Dependent functional linear models with applications to monitoring structural change

Alexander Aue\textsuperscript{1}, Siegfried Hörmann\textsuperscript{2}, Lajos Horváth\textsuperscript{3} and Marie Hušková\textsuperscript{4}

\textsuperscript{1}Department of Statistics, University of California, Davis, One Shields Avenue, Davis, CA 95616, USA
\textsuperscript{2}Département de Mathématique, Université Libre de Bruxelles, CP 215, Boulevard du Triomphe, B-1050 Bruxelles, Belgium
\textsuperscript{3}Department of Mathematics, University of Utah, 155 South 1400 East, Salt Lake City, UT 84112-0090, USA
\textsuperscript{4}Charles University of Prague, Department of Statistics, Sokolovská 83, CZ–186 00 Prague, Czech Republic

Abstract

We study in this paper sequential monitoring procedures that detect instabilities of the regression operator in an underlying (fully) functional regression model allowing for dependence. These open-end and closed-end procedures are built on a functional principal components analysis of both the predictor and response functions, thus giving rise to multivariate detector functions, whose fluctuations are compared against a curved threshold function. The main theoretical result of the paper quantifies the large-sample behavior of the procedures under the null hypothesis of a stable regression operator. To establish these limit results, classical results on functional principal components analysis are generalized to a dependent setting, which may be of interest in its own sake. In an accompanying empirical study we illustrate the finite sample properties, while an application to environmental data highlights practical usefulness. To the best of our knowledge this is the first paper that combines sequential with functional data methodology.

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