Tail Index Estimation for a Filtered Dependent Time Series

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Abstract

We prove Hill’s (1975) tail index estimator is asymptotically normal where the employed data are generated by a stationary parametric time series \( \{x_t(\theta) : t \in \mathbb{Z}\} \) and \( \theta_0 \) is an unknown \( k \times 1 \) vector. We assume \( x_t(\theta_0) \) is unobservable but \( \theta_0 \) is estimable with estimator \( \hat{\theta}_n \) and sample size \( n \geq 1 \), and the filtered series \( x_t(\hat{\theta}_n) \) is observed and used to estimate the tail index. Natural applications include regression residuals, GARCH filters, and weighted sums based on an optimization problem like optimal portfolio selection. Our main result substantially extends Resnick and Starica (1997)’s theory for estimated AR i.i.d. errors and Ling and Peng (2004)’s theory for estimated ARMA i.i.d. errors to a wide range of filtered time series since we do not require \( x_t(\theta_0) \) to be i.i.d., nor generated by a linear process with geometric dependence. We assume \( x_t(\theta_0) \) is \( \beta \)-mixing with possibly hyperbolic dependence, covering ARMA-GARCH filters, ARMA filters with heteroscedastic errors of unknown form, nonlinear filters like threshold autoregressions, and filters based on mis-specified models, as well as i.i.d. errors in an ARMA model. Finally, as opposed to Resnick and Starica (1997) and Ling and Peng (2004) we do not require \( \hat{\theta}_n \) to be super-\( \sqrt{n} \)-convergent when \( x_t(\theta_0) \) has an infinite variance. We allow a far greater variety of plug-ins, including those that are slower than \( \sqrt{n} \), like QML-type estimators for GARCH models.

References


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