Statistica Sinica: Preprint
doi:10.5705/ss.2011.169

Detection with the scan and the average likelihood ratio

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October 2011

Abstract

We investigate the performance of the scan (maximum likelihood ratio statistic) and of the average likelihood ratio statistic in the problem of detecting a deterministic signal with unknown spatial extent in the prototypical univariate sampled data model with white Gaussian noise. Our results show that the scan statistic, a popular tool for detection problems, is optimal only for the detection of signals with the smallest spatial extent. For signals with larger spatial extent the scan is suboptimal, and the power loss can be considerable. In contrast, the average likelihood ratio statistic is optimal for the detection of signals on all scales except the smallest ones, where its performance is only slightly suboptimal. We give rigorous mathematical statements of these results as well as heuristic explanations which suggest that the essence of these findings applies to detection problems quite generally, such as the detection of clusters in models involving densities or intensities or the detection of multivariate signals. We present a modification of the average likelihood ratio that yields optimal detection of signals with arbitrary extent and which has the additional benefit of allowing for a fast computation of the statistic. In contrast, optimal detection with the scan seems to require the use of scale-dependent critical values.

Keywords and phrases. Scan statistic, average likelihood ratio statistic, optimal detection, fast algorithm.

AMS 2000 subject classification. 62G08, 62G10

* Work supported by NUS grant R-155-000-090-112.

** Work supported by NSF grant DMS-1007722 and NIH grant AI077395.