2003 基 礎 圖 論

朱亮儒

國立臺灣師範大學數學系

圖形理論 (Graph Theory) 有著悠久的歷史, 它的起源是十八世紀著名數學家尤拉 (Euler) 爲解決當時流傳的康城七橋問題 (Königsberg Bridge Problem) 而寫的一篇文章. 到了十九世紀英國著名的數學家哈密爾頓 (Hamilton) 所發明的環遊世界遊戲 (A Voyage Round the World — Icosian Game), 對圖形理論提出了一個迄今仍在研究的重要課題. 這些與民間廣泛流傳的難題都有密切的關聯性, 正是圖形論所具有的一大特點, 因而它的發展自然也受到了其他自然科學的影響. 近年來, 圖形理論已成爲處理離散數學模型的一種工具, 也形成組合學研究的重要支柱. 由於圖形理論與日常生活中所遭遇到的問題息息相關, 同時又有廣泛的應用價值, 如: 計算機科學, 物理, 化學與管理科學, 網路分析等, 因此, 圖形理論有其研究的重要價值, 理所當然是各種數學競試中的常客. 本單元將以淺顯的方式介紹圖形的基本理論及相關問題, 例如: 七橋問題, 四色問題, 冒險家問題, 旅行者問題, 推銷員問題, 四立方體問題, 通訊密碼問題, 中國郵差問題, 最短路徑問題, 連通管問題, 朗塞理論 (Ramsey Theory) 等重要的性質, 期能協助研讀者對圖形理論與方法有所了解, 以達到解決日常生活中相關而有趣的問題.

參考書籍:

1. R.J. Wilson & J.J. Watkins, Graphs, An Introductory Approach (歐亞書局).

2. R. Balakrishnan & K. Ranganathan, A Textbook of Graph Theory (俊傑書局).

3. K.R. Parthasarathy, Basic Graph Theory (滄海書局).

4. N. Hartsfield & G. Ringel, Pears in Graph Theory, Academic Press, Inc.

第一節. 基本圖論簡介

圖形 $G(V, E)$, 此處 V 表示頂點集合, E 表示邊集合.
價數 (degree), 迴路 (loop), 重邊 (parallel edge).
價數序列 (degree sequence).
同構圖 (isomorphic graph).
n 階完全圖 (complete graph), 記作 K_n.
完全二部圖 (complete bipartite graph), 記作 $K_{m,n}$.
k- 正則圖 (regular graph), 線圖 (line graph), 對偶圖 (dual graph).
加權圖 (weighted graph), 有向圖 (directed graph), 競賽圖 (tournament).
通路 (walk), 路徑 (trail), 路線 (path), 循環圈 (cycle).
連通圖 (connected graph), 樹 (tree).
Euler 圖, Hamilton 圖, 平面圖 (planar graph).

Euler握手定理：設$G(V,E)$是一個圖，則

$$\sum_{v\in V} d(v) = 2|E| .$$

Graphs serve as mathematical models to analize successfully many concrete real-word problems. Certain problems in physics, chemistry, communications science, computer technology, genetics, psychology, sociology, and linguistics can be formulated as problems in graph theory. Also many branches of mathematics, such as group theory, matrix theory, probablity, and topology, have intractions with graph theory. Some puzzles and various problems of a pratical nature have been instrumental in the development of various topics in graph theory. The famous Königsberg Bridge problem has been the inspiration for the development of Eulerian graph theory. The challenging Hamiltonian graph theory has been developed from the "Around the World" game of Sir William Hamilton. The theory of acyclic graphs was developed for solving problems of electrical networks, and the study of trees was developed for enumerating isomers of organic compounds. The well-known four-color problem has formed the very basis for the development of planarity in graph theory and combinatorial topology. Problems of linear programming and operations research (such as maritime traffic problems) can be tackled by the theory of flows in networks. Kirkman's schoolgirls problem and scheduling problems are examples of problems that can be solved by graph colorings. The study of simplicial complexes can be associated with the study of graph theory. Many more such problems can be added to this list.

第二節. Euler圖

七橋問題，冒險家問題，Euler定理，一筆劃問題，中國郵差問題，最短路徑問題，Fleury方法.

第三節. Hamilton圖

Hamilton謎題("Around the Word" game)，騎士問題，旅行者問題，Dirac定理，Ore定理，Albertson方法，推銷員問題，通訊密碼問題，Redei定理.

第四節. 樹圖與平面圖

樹圖的特徵，連通管問題，Kruskal方法，Prim方法，生成樹的個數，Prüfer數列，Cayley定理，背包(Knapsack)問題，Kuratowski定理，Euler平面圖公式，Euler凸多面體定理.

第五節. 圖形著色

連通數，點著色，邊著色，色彩多項式，Brook定理，König定理，Vizing-Gupta定理，面著色，Heawood五色定理，四色定理，Kirkman Schoolgirls問題，Shannon Switching Game，極端圖與Turan定理，Ramsey理論，完美配對，Hall定理，結婚定理，極大配對與極小覆蓋，König交通網路原理，Menger定理，Whitney定理.