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A BAYESIAN APPROACH TO ENVELOPE QUANTILE REGRESSION

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Abstract: The enveloping approach employs sufficient dimension-reduction techniques to gain estimation efficiency, and has been used in several multivariate analysis contexts. However, its Bayesian development has been sparse, and the only Bayesian envelope construction is in the context of a linear regression. In this paper, we propose a Bayesian envelope approach to a quantile regression, using a general framework that may potentially aid enveloping in other contexts as well. The proposed approach is also extended to accommodate censored data. Data augmentation Markov chain Monte Carlo algorithms are derived for approximate sampling from the posterior distributions. Simulations and data examples are included for illustration.

Key words and phrases: Envelope model, metropolis-within-gibbs sampling, quantile regression, sufficient dimension reduction, tobit quantile.

1. Introduction

The envelope methodology (Cook (2018)) induces a class of models that uses dimension reduction to increase the estimation efficiency in a multivariate analysis, and is sometimes equivalent to taking many additional observations. First proposed for multivariate linear regressions (Cook, Li and Chiaromonte (2010)), it has since been extended to many other contexts, including the partial least squares (PLS) (Cook, Helland and Su (2013); Zhu and Su (2020)), generalized linear models (Cook and Zhang (2015)), elliptical multivariate linear regressions (Forzani and Su (2021)), variable selection (Su et al. (2016)), matrix or tensor variate regressions (Ding and Cook (2018); Li and Zhang (2017)), spatial regressions (Rekabdarkolaee et al. (2020)), and quantile regressions (Ding et al. (2021)). These advances have primarily been made from a frequentist perspective. In practice, there is often a strong motivation to adopt a Bayesian approach. First, a Bayesian approach can incorporate existing knowledge into the

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