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SPACE-TIME ESTIMATION AND PREDICTION UNDER FIXED-DOMAIN ASYMPTOTICS WITH COMPACTLY SUPPORTED COVARIANCE FUNCTIONS

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Abstract: We study the estimation and prediction of Gaussian processes with space-time covariance models belonging to the dynamical generalized Wendland ($\mathcal{DGW}$) family, under fixed-domain asymptotics. Such a class is nonseparable, has dynamical compact supports, and parameterizes differentiability at the origin similarly to the space-time Matérn class.

Our results are presented in two parts. First, we establish the strong consistency and asymptotic normality for the maximum likelihood estimator of the microergodic parameter associated with the $\mathcal{DGW}$ covariance model, under fixed-domain asymptotics. The second part focuses on optimal kriging prediction under the $\mathcal{DGW}$ model and an asymptotically correct estimation of the mean squared error using a misspecified model. Our theoretical results are, in turn, based on the equivalence of Gaussian measures under some given families of space-time covariance functions, where both space or time are compact. The technical results are provided in the online Supplementary material.

Key words and phrases: Fixed-domain asymptotics, microergodic parameter, maximum likelihood, space-time generalized wendland family

1. Introduction

1.1. Context

This study is concerned with fixed-domain asymptotics for the estimation and kriging prediction of Gaussian random fields defined over product spaces $D \times \mathcal{T}$, where D is a subset of $\mathbb{R}^d$ (d is a positive integer) and $\mathcal{T}$ is a compact interval of the real line. The most notable application refers to D as the spatial domain and to $\mathcal{T}$ as time. Although we focus on the space-time case, our results can be analogously applied to the anisotropic spatial case, where the rate of decay in the correlation in one coordinate is different from that of the remaining d coordinates.

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