Introduction to Functional Data Analysis: Mean and Covariance Estimation Revisited

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Abstract

Functional data are data in the form of functions, so they are intrinsically infinite dimensional. However, the sampling plan to observe them might be different. We begin the talk with a brief introduction to functional data by introducing three types of functional data commonly encountered in the literature. Methods to analysis functional data are termed "Functional Data Analysis", abbreviated as FDA, and they are often nonparametric.

We next consider nonparametric estimation of the mean and covariance functions and investigate the performance of local linear smoothers for both the mean and covariance functions with a general weighing scheme, which includes two commonly used schemes, equal weight per observation (OBS), and equal weight per subject (SUBJ), as two special cases. We provide a comprehensive analysis of their asymptotic properties on a unified platform for all types of sampling plan. Three types of asymptotic properties are investigated in this paper: asymptotic normality, L 2 convergence and uniform convergence. The asymptotic theories are unified on two aspects: (1) the weighing scheme is very general; (2) the magnitude of the number Ni of measurements for the ith subject relative to the sample size n can vary freely. Based on the relative order of Ni to n, functional data are partitioned into three new types: non-dense, dense, and ultra-dense functional data for the OBS and SUBJ schemes. These two weighing schemes are compared theoretically and numerically.

In reality, however, it is difficult to know which weighing scheme is best for the data in hand, we thus propose an optimal weighing scheme that automatically adapts to the sampling plan of the functional data.