Principal component analysis for time series

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Abstract

We extend the principal component analysis (PCA) to second-order stationary vector time series in the sense that we seek for a contemporaneous linear transformation for a $p$-variate time series such that the transformed series is segmented into several lower-dimensional subseries, and those subseries are uncorrelated with each other both contemporaneously and serially. Therefore those lower-dimensional series can be analysed separately as far as the linear dynamic structure is concerned. Technically it boils down to an eigenanalysis for a positive definite matrix. When $p$ is large, an additional step is required to perform a permutation in terms of either maximum cross-correlations or FDR based on multiple tests. The asymptotic theory is established for both fixed $p$ and diverging $p$ when the sample size $n$ tends to infinity. Numerical experiments with both simulated and real datasets indicate that the proposed method is an effective initial step in analysing multiple time series data, which leads to substantial dimension reduction in modelling and forecasting high-dimensional linear dynamical structures. Unlike PCA for independent data, there is no guarantee that the required linear transformation exists. When it does not, the proposed method provides an approximate segmentation which leads to the advantages in, for example, forecasting for future values. The method can also be adapted to segment multiple volatility processes.

(Joint work with Jinyuan Chang and Bin Guo)