Kernel-based estimators, functional predictors and nonparametric regression

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Technologies progress in terms of computational tools or memory capicities allow to handle larger and larger datasets. At the same time, monitoring devices, electronic equipments like sensors (for registrering images, temperatures, gas contents,...) become more and more sophisticated. This high-tech evolution offers the opportunity to observe phenomena in a more and more accurate way by producing statistical units sampled over a finer and finer grid. Such high-tech data are commonly named "functional data" and are collected in numerous domains: biomechanics (e.g. human movements), chemometrics (spectrometric curves), econometrics (e.g. stock market indices), geophysics (e.g. spatio-temporal, time series of satellite images), medicine (e.g. electro-cardiograms, electro-encephalograms).

These new kind of data involves functional aspects and there is really a need of developing specific functional statistical methods. In the 1990's, linear statistical methods (see Ramsay and Silverman, 2005) were first extended to the functional case (functional principal component, linear functional regression, functional analysis of variance, functional discriminant analysis,...). From the begining of the 2000's, various nonlinear statistical methods able to take into account functional data were developed (see Ferraty and Vieu, 2006). The aim of this work is to present recent advances on kernel-based methods in nonparametric regression model (i.e. $Y = r(X) + \varepsilon$ with $r(.)$ continuous or lipschitzian,...) dealing with functional predictor (i.e. X random variable valued in some infinite-dimensional space). After introducing the problematic via examples, we recall the definition of what we name *nonparametric functional regression* and also of the kernel estimator of the regression operator ($r(.)$). An overview on basic theoretical properties (pointwise rate, uniform rate, asymptotic normality) of the kernel estimator are given. Afterthen, we propose a scope of related useful methods recently investigated

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in the context of nonparametric functional regression: cross-validation, bootstrap, kNN-estimator and structural test. At last, we propose to point out some interesting prospects oriented towards semiparametric modelling and nonparametric selection-variable-method in high-dimensional setting.

Relevant books

Ferraty, F., Vieu, P. (2006). *NonParametric Functional Data Analysis: Theory and Practice.* Springer Series in Statistics. Springer, New York.

Ferraty, F., Romain, Y. (2010). *Handbook on Statistics and Functional Data Analysis* (Eds). Oxford University Press (forthcoming book).

Ramsay, J.O., Silverman, B.W. (2005): *Functional data analysis.* Second edition. Springer Series in Statistics. Springer, New York.