An Asymptotically Optimal Change Detection Strategy Under Non-traditional Global False Alarm Probability Constraint

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In nineteen sixties Shiryaev developed Bayesian theory of change-point detection for independent and identically distributed (i.i.d.) sequences. In Shiryaev's setting, which will be referred to as the classical setting, the goal is to minimize an average detection delay under the constraint imposed on the average probability of false alarm. Recently, Tartakovsky and Veeravalli [*Theory Probab. Appl.* **49**, no. 3, pp. 458-497, 2005] developed a general Bayesian asymptotic change-point detection theory that is not limited to a restrictive i.i.d. assumption. It was proved that Shiryaev's classical detection procedure is asymptotically optimal under traditional average false alarm probability constraint, assuming that this probability is small, for general non-i.i.d. data models and almost arbitrary prior distributions. In the present paper, we consider a less conventional approach where the constraint is imposed on the global, supremum false alarm probability. An asymptotically optimal Bayesian change detection procedure is proposed and thoroughly evaluated for both i.i.d. and non-i.i.d. models as the global false alarm probability approaches zero.
More specifically, consider the class of stopping times $\Delta_\alpha = \{\tau : P_\infty(\tau < \infty) \le \alpha\}$ for which the worst-case (global) false alarm probability $\sup_{k\ge1} P_k(\tau < k) = P_\infty(\tau < \infty)$ is restricted by the given number $\alpha < 1$, where P_k is the probability measure for the fixed point of change $\lambda = k$. Let $E^\pi(\cdot) = \sum_{k=0}^\infty E_k(\cdot)\pi_k$, where E_k is the operator of expectation for $\lambda = k$ and $\pi_k = P(\lambda = k)$ is the prior distribution of the change point λ, which can be arbitrary, not necessarily geometric. We propose an asymptotically optimal solution to the following optimization problem: $\inf_{\tau\in\Delta_\alpha} E^\pi(\tau-\lambda)^+ \to \tau_{\rm opt}$ letting α go to zero. Moreover, we address the problem of minimizing higher moments of the detection delay $\inf_{\tau\in\Delta_\alpha} E^\pi[(\tau-\lambda)^m|\tau \ge \lambda]$ for any $m > 1$ as $\alpha \to 0$. The asymptotic optimality results hold not only for i.i.d. data models but also for general stochastic models whenever a log-likelihood ratio process for prechange and postchange hypotheses obeys the strong law of large numbers with a certain rate of convergence.

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